[2]Cobleigh, B. R., "High-Angle-of-Attack Yawing Moment Asymmetry of the X-31 Aircraft from Flight Test," NASA CR 186030, Sept. 1994.

[3]Komerath, N. M., Liou, S.-G., DeBry, B., and Lenakos, J., "Measurements of the Unsteady Vortex Flow over a Wing-Body at Angle of Attack," AIAA Paper 92-2729, June 1992.

[4]Lee, R., Hanff, E. S., and Kind, R. J., "Wind-Tunnel Investigation of Dynamic Manipulation of Forebody Vortices," AIAA Paper 95-1794, June 1995.

[5]Williams, D., and Bernhardt, J., "Proportional Control of Asymmetric Forebody Vortices with the Unsteady Bleed Technique," AIAA Paper 90-1629, June 1990.

[6]Malcolm, G. N., "Forebody Vortex Control," AGARD Rept. 776, March 1991, pp. 1-6–40.

[7]Moskovitz, C., Hall, R., and Dejarnette, F., "Experimental Investigation of a New Device to Control the Asymmetric Flowfield on Forebodies at Large Angles of Attack," AIAA Paper 90-0068, Jan. 1990.

[8]Bridges, D. H., and Hornung, H. G., "Elliptic Tip Effects on the Vortex Wake of an Axisymmetric Body at Incidence," *AIAA Journal*, Vol. 32, No. 7, 1992, pp. 1437–1445.

[9]Degani, D., and Tobak, M., "Numerical Simulation of Upstream Disturbance on Flows Around a Slender Body," AIAA Paper 93-2956, June 1993.

[10]Darden, L. A., and Komerath, N. M., "Forebody Vortex Control at High Incidence Using a Moveable Nose Stagnation Point," AIAA Paper 95-1775, June 1995.

[11]Bendat, J. S., and Piersol, A. G., *Random Data Analysis and Measurement Procedures*, 2nd ed., Wiley, New York, 1986, Chaps. 5, 6.

[12]Darden, L. A., Peterson, K. G., and Komerath, N. M., "Vortex Control Using a Moveable Nose with Pressure Feedback," AIAA Paper 95-3468, Aug. 1995.

[13]Peterson, K. G., Darden, L. A., and Komerath, N. M., "Dynamic Roll Control Experiments Using a Moveable Nosetip," AIAA Paper 96-0789, Jan. 1996.

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Piezoelectric Constitutive Equations for a Plate Shape Sensor/Actuator

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Introduction

USING piezoelectric sensors and actuators for monitoring and controlling the motion of a flexible structure has been widely discussed during the past few decades.[1,2] The flexible structures concerned are laminated plates, and then the piezoelectric sensors and actuators used are of plate shape. Therefore, when one analyzes and simulates the mechanical and electrical motions of a structure based on the classical laminate plate theory, the plane form of the general piezoelectric constitutive equations, i.e., the piezoelectric constitutive equations of a plate shape sensor or actuator, should be adopted. In fact, related previous reports have attempted to do so.[3–6] However, in previous reports the plane form piezoelectric constitutive equations are only written in forms. The relationships between the plane form piezoelectric constitutive equations and the general piezoelectric constitutive equations have not been shown. These relationships, according to which the constants in the plane form equations can be obtained from the manufacturer-given constants in the general equations, are necessary and important when one carries out analysis of laminated piezoelectric plates. In this Note, from the general piezoelectric constitutive equations and based on the Kirchhoff hypothesis of the classical laminate plate theory, we will derive the piezoelectric constitutive equations of a plate shape sensor or actuator.

Equations

The commonly named piezoelectric constitutive equations, i.e., the so-called general piezoelectric constitutive equations in this Note, can be written as[7]

$$D_i = e_{ij} S_j + \varepsilon^S_{ik} E_k \tag{1a}$$

$$T_l = c^E_{lj} S_j - e_{kl} E_k \tag{1b}$$

where $i, k = 1, 2, 3$ and $j, l = 1, 2, 3, 4, 5, 6$; D_i and E_k represent the electric field and electric displacement, respectively; S_j and T_l represent strain and stress, respectively; e_{ij} represents the piezoelectric stress/charge matrix, ε^S_{ik} represents the permitivity matrix measured at constant strain, and c^E_{lj} represents the elastic stiffness matrix measured at constant electric field.

Usually, the thickness direction of a plate shape sensor or actuator is designated by direction 3. Its two surfaces that are parallel to the plate plane are the two electrodes. In classical laminate plate theory, the Kirchhoff hypothesis is adopted.[8] That is, T_3 can be neglected relative to T_1 and T_2; any line perpendicular to the plate midplane before deformation remains perpendicular to the midplane after deformation; then there are no shear strains in any plane perpendicular to the plate midplane. The result is

$$T_3 = 0, \qquad S_4 = S_5 = 0 \tag{2}$$

Substituting Eq. (2) into Eq. (1b), we can obtain

$$T_1 = c^E_{11} S_1 + c^E_{12} S_2 + c^E_{13} S_3 + c^E_{16} S_6 - e_{11} E_1 - e_{21} E_2 - e_{31} E_3 \tag{3a}$$

$$T_2 = c^E_{12} S_1 + c^E_{22} S_2 + c^E_{23} S_3 + c^E_{26} S_6 - e_{12} E_1 - e_{22} E_2 - e_{32} E_3 \tag{3b}$$

$$0 = c^E_{13} S_1 + c^E_{23} S_2 + c^E_{33} S_3 + c^E_{36} S_6 - e_{13} E_1 - e_{23} E_2 - e_{33} E_3 \tag{3c}$$

$$T_6 = c^E_{16} S_1 + c^E_{26} S_2 + c^E_{36} S_3 + c^E_{66} S_6 - e_{16} E_1 - e_{26} E_2 - e_{36} E_3 \tag{3d}$$

According to Eq. (3c), we get

$$S_3 = -\frac{c^E_{13} S_1 + c^E_{23} S_2 + c^E_{36} S_6 - e_{13} E_1 - e_{23} E_2 - e_{33} E_3}{c^E_{33}} \tag{4}$$

Substituting Eq. (4) into Eqs. (3a), (3b), and (3d) we get

$$\begin{aligned}
T_1 &= \left[c^E_{11} - \left(c^E_{13}\right)^2 / c^E_{33}\right] S_1 + \left(c^E_{12} - c^E_{13} c^E_{23} / c^E_{33}\right) S_2 \\
&\quad + \left(c^E_{16} - c^E_{13} c^E_{36} / c^E_{33}\right) S_6 - \left(e_{11} - e_{13} c^E_{13} / c^E_{33}\right) E_1 \\
&\quad - \left(e_{21} - e_{23} c^E_{13} / c^E_{33}\right) E_2 - \left(e_{31} - e_{33} c^E_{13} / c^E_{33}\right) E_3 \\
T_2 &= \left(c^E_{12} - c^E_{13} c^E_{23} / c^E_{33}\right) S_1 + \left[c^E_{22} - \left(c^E_{23}\right)^2 / c^E_{33}\right] S_2 \\
&\quad + \left(c^E_{26} - c^E_{23} c^E_{36} / c^E_{33}\right) S_6 - \left(e_{12} - e_{13} c^E_{23} / c^E_{33}\right) E_1 \\
&\quad - \left(e_{22} - e_{23} c^E_{23} / c^E_{33}\right) E_2 - \left(e_{32} - e_{33} c^E_{23} / c^E_{33}\right) E_3 \\
T_6 &= \left(c^E_{16} - c^E_{13} c^E_{36} / c^E_{33}\right) S_1 + \left(c^E_{26} - c^E_{23} c^E_{36} / c^E_{33}\right) S_2 \\
&\quad + \left[c^E_{66} - \left(c^E_{36}\right)^2 / c^E_{33}\right] S_6 - \left(e_{16} - e_{13} c^E_{36} / c^E_{33}\right) E_1 \\
&\quad - \left(e_{26} - e_{23} c^E_{36} / c^E_{33}\right) E_2 - \left(e_{36} - e_{33} c^E_{23} / c^E_{36}\right) E_3
\end{aligned} \tag{5}$$

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Substituting Eqs. (2) and (4) into Eq. (1a), we can obtain

$$
\begin{aligned}
D_1 &= \left(e_{11} - e_{13}c_{13}^E/c_{33}^E\right)S_1 + \left(e_{12} - e_{13}c_{23}^E/c_{33}^E\right)S_2 \\
&\quad + \left(e_{16} - e_{13}c_{36}^E/c_{33}^E\right)S_6 + \left[\varepsilon_{11}^S + (e_{13})^2/c_{33}^E\right]E_1 \\
&\quad + \left(\varepsilon_{12}^S + e_{13}e_{23}/c_{33}^E\right)E_2 + \left(\varepsilon_{13}^S + e_{13}e_{33}/c_{33}^E\right)E_3 \\
D_2 &= \left(e_{21} - e_{23}c_{13}^E/c_{33}^E\right)S_1 + \left(e_{22} - e_{23}c_{23}^E/c_{33}^E\right)S_2 \\
&\quad + \left(e_{26} - e_{23}c_{36}^E/c_{33}^E\right)S_6 + \left(\varepsilon_{12}^S + e_{13}e_{23}/c_{33}^E\right)E_1 \\
&\quad + \left[\varepsilon_{22}^S + (e_{23})^2/c_{33}^E\right]E_2 + \left(\varepsilon_{23}^S + e_{23}e_{33}/c_{33}^E\right)E_3 \\
D_3 &= \left(e_{31} - e_{33}c_{13}^E/c_{33}^E\right)S_1 + \left(e_{32} - e_{33}c_{23}^E/c_{33}^E\right)S_2 \\
&\quad + \left(e_{36} - e_{33}c_{36}^E/c_{33}^E\right)S_6 + \left(\varepsilon_{13}^S + e_{13}e_{33}/c_{33}^E\right)E_1 \\
&\quad + \left(\varepsilon_{23}^S + e_{23}e_{33}/c_{33}^E\right)E_2 + \left[\varepsilon_{33}^S + (e_{33})^2/c_{33}^E\right]E_3
\end{aligned} \tag{6}
$$

Equations (5) and (6) can be written as the following forms that are the plane form piezoelectric constitutive equations:

$$
\begin{bmatrix} T_1 \\ T_2 \\ T_6 \end{bmatrix} = \begin{bmatrix} c_{11}^X & c_{12}^X & c_{16}^X \\ c_{12}^X & c_{22}^X & c_{26}^X \\ c_{16}^X & c_{26}^X & c_{66}^X \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \\ S_6 \end{bmatrix} - \begin{bmatrix} e_{11}^X & e_{21}^X & e_{31}^X \\ e_{12}^X & e_{22}^X & e_{32}^X \\ e_{16}^X & e_{26}^X & e_{36}^X \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} \tag{7a}
$$

$$
\begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \begin{bmatrix} e_{11}^X & e_{12}^X & e_{16}^X \\ e_{21}^X & e_{22}^X & e_{26}^X \\ e_{31}^X & e_{32}^X & e_{36}^X \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \\ S_6 \end{bmatrix} + \begin{bmatrix} \varepsilon_{11}^X & \varepsilon_{12}^X & \varepsilon_{13}^X \\ \varepsilon_{12}^X & \varepsilon_{22}^X & \varepsilon_{23}^X \\ \varepsilon_{13}^X & \varepsilon_{23}^X & \varepsilon_{33}^X \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} \tag{7b}
$$

where

$$
c_{ij}^X = c_{ij}^E - c_{i3}^E c_{j3}^E/c_{33}^E, \qquad e_{kj}^X = e_{kj} - e_{k3}c_{j3}^E/c_{33}^E
$$
$$
\varepsilon_{kl}^X = \varepsilon_{kl}^S + e_{k3}e_{l3}/c_{33}^E \tag{8}
$$

are the corresponding new elastic stiffness matrix, piezoelectric stress/charge matrix, and permittivity matrix, respectively; i, $j = 1$, 2, 6 and k, $l = 1$, 2, 3, and the superscript X denotes the constants in the new equations.

To date, the piezoelectric materials of a commonly discussed and used sensor/actuator are polyvinylidene fluoride polymer (PVDF) or PZT (lead, zirconate, titanate), etc., which are at least orthotropic. The pole direction of such a sensor/actuator is in its thickness direction. In addition, the electric field in the sensor/actuator can be treated as a uniform electric field. Then among the components of the electric field and electric displacement the possible nonzero components are E_3 and D_3. For later uses, according to Eqs. (7) and (8) and their concrete material constants, we write the piezoelectric constitutive equations for an orthotropic plate shape sensor/actuator:

$$
\begin{bmatrix} T_1 \\ T_2 \\ T_6 \end{bmatrix} = [c] \begin{bmatrix} S_1 \\ S_2 \\ S_6 \end{bmatrix} - E_3 \begin{bmatrix} e_{31} - \left(c_{13}^E/c_{33}^E\right)e_{33} \\ e_{32} - \left(c_{23}^E/c_{33}^E\right)e_{33} \\ 0 \end{bmatrix} \tag{9a}
$$

$$
D_3 = e_{31}^X S_1 + e_{32}^X S_2 + e_{36}^X S_6 + \varepsilon_{33}^X E_3 \tag{9b}
$$

where

$$
[c] = \begin{bmatrix} c_{11}^E - \left(c_{13}^E\right)^2/c_{33}^E & c_{12}^E - c_{13}^E c_{23}^E/c_{33}^E & 0 \\ c_{12}^E - c_{13}^E c_{23}^E/c_{33}^E & c_{22}^E - \left(c_{23}^E\right)^2/c_{33}^E & 0 \\ 0 & 0 & c_{66}^E \end{bmatrix}
$$
$$
e_{31}^X = e_{31} - \left(c_{13}^E/c_{33}^E\right)e_{33}, \qquad e_{32}^X = e_{32} - \left(c_{23}^E/c_{33}^E\right)e_{33} \tag{10}
$$
$$
\varepsilon_{33}^X = \varepsilon_{33}^S + \left(e_{33}^2/c_{33}^E\right)
$$

Conclusions

We have given the exact formulas of the piezoelectric constitutive equations for a plate shape sensor/actuator, which are complements of the theory of laminated piezoelectric plates. According to our formulas, the constants in the new equations can be obtained directly from those in the general piezoelectric constitutive equations, whereas the latter are given by the manufacturer or can be directly calculated from the manufacturer-given constants according to the existing knowledge in piezoelectricity. The other three forms of the plane form piezoelectric constitutive equations corresponding to those of the general piezoelectric constitutive equations can be obtained from our equations through simple algebraic calculations. Our results can be used when one analyzes the motion of a laminated piezoelectric plate.

References

[1] Rao, S. S., and Sunar, M., "Piezoelectricity and Its Use in Disturbance Sensing and Control of Flexible Structures: A Survey," *Applied Mechanics Review*, Vol. 47, No. 4, 1994, pp. 113–123.

[2] Sunar, M., and Rao, S. S., "Distributed Modeling and Actuator Location for Piezoelectric Control Systems," *AIAA Journal*, Vol. 34, No. 10, 1996, pp. 2209–2218.

[3] Lee, C. K., "Theory of Laminated Piezoelectric Plates for the Design of Distributed Sensors/Actuators. Part I: Governing Equations and Reciprocal Relationships," *Journal of the Acoustical Society of America*, Vol. 87, No. 3, 1990, pp. 1144–1158.

[4] Wang, B. T., and Rogers, C. A., "Laminate Plate Theory for Spatially Distributed Induced Strain Actuators," *Journal of Composite Materials*, Vol. 25, 1991, pp. 433–452.

[5] Hwang, W.-S., and Park, H. C., "Finite Element Modeling of Piezoelectric Sensors and Actuators," *AIAA Journal*, Vol. 31, No. 5, 1993, pp. 930–937.

[6] Miller, S. E., Oshman, Y., and Abramovich, H., "Modal Control of Piezoelectric Anisotropic Rectangular Plates, Part 1: Modal Transducer Theory," *AIAA Journal*, Vol. 34, No. 9, 1996, pp. 1868–1875.

[7] Tiersten, H. F., *Linear Piezoelectric Plate Vibrations*, Plenum, New York, 1969.

[8] Tsai, S. W., and Hahn, H. T., *Introduction to Composite Materials*, Technomic, Lancaster, PA, 1980.

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Dispersion of Axisymmetric Elastic Waves in Thick-Walled Orthotropic Pipes

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Introduction

A METHOD for studying harmonic wave propagation in thick orthotropic cylinders of infinite length was presented by Markuš and Mead.[1] Displacements were modeled by trigonometric functions and Frobenius power series. Harmonic wave propagation in shells and rods of infinite length was studied by Mazúch.[2] Relations for a finite element model were derived for general anisotropy of a linearly elastic material. The generalized eigenvalue problem was solved by the Lanczos method with simple orthogonalization by Mazúch.[3] Jing and Tzeng[4] presented an approximate elasticity solution for arbitrarily laminated anisotropic cylindrical closed shells of finite length with simply supported ends.

In this Note, a modification of the last method is used to obtain dispersion curves for the axisymmetric problem of arbitrarily laminated, orthotropic, unclosed, cylindrical pipes of infinite length. The first five dispersion curves are obtained. A convergence study is performed for a single orthotropic layer with the elastic moduli

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